Billiken Challenge Solutions
January 2018

1. Suppose you flip a coin until you get four heads in a row. What is your expected number of flips?

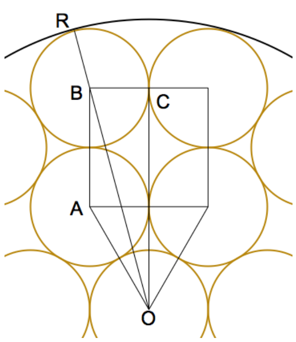
Solution: Let ν be the expected number of flips. Let μ be the expected number of flips for the next occurrence of HHHH after just having gotten HHHH. Because in the long run, HHHH occurs in any spot with probability $1/16$, $\mu = 16$. On the other hand, after HHHH occurs there are two possibilities: Heads results in another occurrence of HHHH immediately, while Tails results in an expected $1 + \nu$ flips before another HHHH occurs. So,

$$\mu = \frac{1}{2} \cdot 1 + \frac{1}{2}(1 + \nu)$$

and solving for ν gives $\nu = 30$.

This problem and its solution were suggested by Prof. Darrin Speegle.

2. Find the radius of the outer circle when 19 unit circles are packed inside it as shown.



Solution: Since BR is a radius of the circle at B, it is perpendicular to the big circle. The line OR is a radius of the big circle, so it is perpendicular to the big circle. Then B is actually on the line OR as shown.

All inner circles have radius 1. From the Pythagorean theorem, $OC = 2 + \sqrt{3}$. Since $BC = 1$, $OB = \sqrt{1^2 + (2 + \sqrt{3})^2} = \sqrt{8 + 4\sqrt{3}}$. Then $OR = 1 + \sqrt{8 + 4\sqrt{3}} \approx 4.86$.

That this is the optimal packing of 19 circles inside another circle was proved by Fodor in 1999. For more information on circle packings, try `http://www.packomania.com`.