Derivatives II: Exponential and Logarithmic Functions.

We will find the derivatives of the other elementary functions.
(This corresponds to section 3.2 in our book)

1. Exponential Functions.
Let \( f(x) = a^x \), where \( a \) is any positive constant (but not 0). The derivative of this function, according to the definition, will be the following function:

\[
\frac{d}{dx}a^x = \lim_{h \to 0} \frac{a^{x+h} - a^x}{h} = \lim_{h \to 0} \frac{a^x(a^h - 1)}{h} = a^x \lim_{h \to 0} \left( \frac{a^h - 1}{h} \right)
\]

The question then becomes, what is \( \lim_{h \to 0} \left( \frac{a^h - 1}{h} \right) \)? Below we will argue that this is a constant which depends on \( a \). So we get that: \( \frac{d}{dx}a^x = K \cdot a^x \), where \( K \) is some constant.

In your calculator, enter the functions \( y = \frac{a^x - 1}{x} \) using \( a = 2, e, 3 \) and 5.

Complete the following table:

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<th>x</th>
<th>( y = \frac{(2^x - 1)}{x} )</th>
<th>( y = \frac{(e^x - 1)}{x} )</th>
<th>( y = \frac{(3^x - 1)}{x} )</th>
<th>( y = \frac{(5^x - 1)}{x} )</th>
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<td>( \lim_{h \to 0} \left( \frac{a^h - 1}{h} \right) ) = ?</td>
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Compute: \( \ln(2) = \) \( \ln(e) = \) \( \ln(3) = \) \( \ln(5) = \)

Write down the formula for the following derivatives:

\( \frac{d}{dx}a^x = \) \( \frac{d}{dx}e^x = \)

We have the following special case:
2. Logarithmic functions.

*We will restrict our attention to the natural logarithm.* To prove the derivative formula for the function $y = \ln(x)$ we need “heavier machinery” than we have available at this point. So we will just state the formula without proof for now.

$$\frac{d}{dx} \ln(x) = \frac{1}{x}$$

Find the following derivatives:

$$\frac{d}{dx} \left( e^x + (2.5)^x \right) =$$

$$\frac{d}{dx} \left( 5e^x + 3 \left( \frac{1}{2} \right)^x \right) =$$

$$\frac{d}{dx} \left( x^2 + \ln(x) \right) =$$

$$\frac{d}{dx} \left( \frac{1}{3} x^{27} - 2\ln(x) \right) =$$