Section 14 The Order Topology

**Definition:** Recall that an order relation “<” is a relation with the following properties:
1. If x ≠ y, then either x < y or y < x (Comparability)
2. If x < y then x ≠ y (Non-reflexivity)
3. If x < y and y < z, then x < z (Transitivity)

(pg 24 – 25)

**Definition:** Let X be a set with a simple order relation; Assume X has more than one element.

A basis B for the order topology on X is given by the collection of all sets of these types:
1. All open intervals (a, b) in X
2. All intervals of the form [a₀, b), where a₀ is the smallest element (if any) of X
3. All intervals of the form [a, b₀), where b₀ is the largest element (if any) of X

EX 1. The standard topology on the real numbers is the order topology derived from the usual order on the real numbers.

EX2. Order topology on R x R induced by the dictionary order. What do basis elements look like?

**Definition:** open and closed rays:
(a, ∞) = \{x \mid x > a\}
(−∞, a) = \{x \mid x < a\}
[a, ∞) = \{x \mid x ≥ a\}
(−∞, a] = \{x \mid x ≤ a\}