1. Cycloid - the path of a point on the perimeter of a circle rolling along a plane. General equation: \( x = a(t - \sin(t)), \quad y = a(1 - \cos(t)) \), where \( a \) is the radius of the rolling circle.

With your calculator in RADIANT and PARAMETRIC mode, graph \( X_{1T} = T - \sin(T), \quad Y_{1T} = 1 - \cos(T) \), using the window \( T \in [0, 2\pi, \frac{\pi}{24}], \quad X \in [0, 13, 1], \quad Y \in [0, 4, 1] \). Use ZSQUARE after the graph is sketched. The curve shown is the path traced by a point on a rolling circle. Change TMAX to \( 4\pi \) to see the path for two revolutions of the circle.

Sketch what you see here:

### Spirograph curves

2. Hypocycloid - the path of a point on the perimeter of a circle which is rolling around the inside of a fixed circle. General equation: \( x = (a - b) \cos(t) + b \cos(\frac{a - b}{b}t), \quad y = (a - b) \sin(t) - b \sin(\frac{a - b}{b}t) \), where \( a \) is the radius of the fixed circle, and \( b \) is the radius of the rolling circle.

Graph \( X_{1T} = 1.5 \cos(T) + 0.5 \cos(3T), \quad Y_{1T} = 1.5 \sin(T) - 0.5 \sin(3T) \) for \( T \in [0, 2\pi, \frac{\pi}{24}] \) in the ZDECIMAL window. (Here \( a = 2, b = 0.5 \))

Next graph \( X_{1T} = \cos(T) + 1.5 \cos(\frac{2T}{3}), \quad Y_{1T} = \sin(T) - 1.5 \sin(\frac{2T}{3}) \) for \( T \in [0, 6\pi, \frac{\pi}{12}] \). (Here \( a = 2, b = 1.5, \) and the small circle needs to go around the inside of the fixed circle three times to close the curve.)

Sketch both curves here:

3. Epicycloid - the path of a point on the perimeter of a circle which is rolling around the outside of a fixed circle. General equation: \( x = (a + b) \cos(t) - b \cos(\frac{a + b}{b}t), \quad y = (a + b) \sin(t) - b \sin(\frac{a + b}{b}t) \), where \( a \) is the radius of the fixed circle, and \( b \) is the radius of the rolling circle.

Graph \( X_{1T} = 2 \cos(T) - 0.5 \cos(4T), \quad Y_{1T} = 2 \sin(T) - 0.5 \sin(4T) \) for \( T \in [0, 2\pi, \frac{\pi}{24}] \) in the ZDECIMAL window. (Here \( a = 1.5, b = 0.5 \))

Next graph \( X_{1T} = 2.2 \cos(T) - 0.8 \cos(\frac{11T}{4}), \quad Y_{1T} = 2.2 \sin(T) - 0.8 \sin(\frac{11T}{4}) \) for \( T \in [0, 8\pi, \frac{\pi}{12}] \). (Here \( a = 1.4, b = 0.8, \) and the small circle needs to go around the outside of the fixed circle four times to close the curve.)

Sketch both curves here: