Exercises

Chapter 7.6 # 1, 4, 9 (beware that the answers in the back of the book are wrong for many of the 7.6 problems, including #9)

Chapter 7.7 # 1, 2, 3, 9, 15

Chapter 8.1 # 1, 3

Problem A: Let $A$ be the matrix from Section 7.7 Exercise 9. Solve $Av = b$ when $b$ is each of the following vectors:

$$b_1 = \begin{pmatrix} 4 \\ 0 \\ 12 \end{pmatrix}; \quad b_2 = \begin{pmatrix} 0 \\ 12 \\ 0 \end{pmatrix}; \quad b_3 = \begin{pmatrix} -12 \\ 24 \\ -12 \end{pmatrix}$$

Problem B: Let $R_\theta = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$ be a rotation matrix. Show that $R_\theta^{-1} = R_{-\theta}$.

Problem C: A matrix $A$ is called orthogonal if $A^t = A^{-1}$. Show that if $A$ is orthogonal then $AA^t = I$ and $A^tA = I$. Show that if $A$ is orthogonal then $A^t$ is orthogonal.

Problem D: Show that $R_\theta$ is orthogonal.

Problem E: Suppose $A$ and $B$ are the same size, and suppose that both $A$ and $B$ are orthogonal. Show that $AB$ is orthogonal.

Problem F: Suppose $A$ and $B$ are the same size. If $A$ is symmetric and $B$ is orthogonal, show that $BAB^{-1}$ is symmetric.

Problem G: Suppose $A$ is orthogonal. Show that the column vectors of $A$ are orthogonal to each other and that each column vector of $A$ has magnitude 1.

Hint: The product of two matrices has entries which are dot products of rows of one matrix with columns of the other.

Problem H: Suppose $A$ is orthogonal. Show that multiplication by $A$ preserves lengths. That is, for any vector $v$, show that $||Av|| = ||v||$.

Hint: For a column vector $w$, you can compute $w \cdot w = w^t w$. 