Math 1520 – Sample Exam 1

You may use a graphing calculator (TI-83, 84, for example) on this exam, but not one that can perform symbolic integration (TI-89, for example).

There are 7 questions, worth a total of 100 points.

1. (40) Calculate four of these integrals (do not do all five - only your first four will be graded):

   \[ A = \int \frac{x^2 + x + 1}{x + 1} \, dx \quad B = \int \frac{\sqrt{\ln(x + 1)}}{x + 1} \, dx \quad C = \int x \cos(3x) \, dx \]

   \[ D = \int \frac{e^x}{1 + e^{2x}} \, dx \quad E = \int \frac{1}{x^2 + 5x + 6} \, dx \]

2. (10) True or false:

   (a) \[ \int \frac{1}{x} \, dx = x^0 + C \]

   (b) \[ \int \frac{1}{x} \, dx = x^{-1} + C \]

   (c) \[ \int \frac{1}{x} \, dx = \ln |x| + C \]

   (d) \[ \int \frac{1}{x} \, dx = \ln |2x| + C \]

   (e) \[ \int \frac{1}{x} \, dx = x \ln |x| - x + C \]

3. (10) Let \( \text{Li}(x) = \int \frac{dx}{\ln(x)} \). Calculate \( \int \ln(\ln(x)) \, dx \) in terms of \( \text{Li}(x) \).

4. (10) Compute \( \int_2^\infty \frac{3}{(x - 1)^{3/2}} \, dx \), if the integral converges.

5. (10) For which values of \( p \) does the integral \( \int_0^\infty \frac{dx}{\sqrt{x^p + 1}} \) converge? Justify your answer.

6. (10) The picture below shows the graph of \( \cos(x) \) for \( -\frac{\pi}{2} \leq x \leq \frac{\pi}{2} \).
On the picture, draw rectangles corresponding to the left approximation $\text{LEFT}(6)$.
Will $\text{LEFT}(6)$ over- or under-approximate the area under the curve? (Look carefully and think!)

(10) 7. (a) Compute $I = \int_0^1 x^2 dx$.
    (b) Compute the trapezoid approximation $\text{TRAP}(2)$ for $I$.
    (c) Compute the midpoint approximation $\text{MID}(2)$ for $I$.
    (d) Compute the Simpson’s approximation $\text{SIMP}(2) = \frac{2\text{MID}(2) + \text{TRAP}(2)}{3}$
    (e) Explain why $\text{SIMP}(2)$ gives the exact value of the integral $I$. 