The Fourier series for \( f(x) \) defined on \([-\pi, \pi]\) is

\[
f(x) = a_0 + a_1 \cos(x) + a_2 \cos(2x) + a_3 \cos(3x) + a_4 \cos(4x) + \cdots
+ b_1 \sin(x) + b_2 \sin(2x) + b_3 \sin(3x) + b_4 \sin(4x) + \cdots
\]

where

\[
a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx, \quad a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx, \quad b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx
\]

Note that for even functions, only cosine terms are non-zero, and for odd functions only sine terms are non-zero.

Let \( f(x) = x^2 \) on the interval \([-\pi, \pi]\). Since \( f \) is even, the Fourier series for \( f(x) \) will have only \( a_0 \) and cosine terms.

1. Find the value of \( a_0 \).

2. Find the value of \( a_n \) for \( n > 0 \). Use scratch paper and/or get computer assistance.

3. Write the Fourier series for \( x^2 \) on \([-\pi, \pi]\):

\[
x^2 = \]

4. Graph the first few terms of the Fourier series and see how it converges to \( x^2 \).

5. Solve the “Basel Problem”: Find the value of the series \( 1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \frac{1}{36} + \cdots \)

   To do this, plug in \( x = \pi \) to your equation in problem 3 and simplify.