1. Sketch the curve given by $x = t^2, y = t^3$ for $t \in [0, 2]$. Find its arclength exactly.

**Solution:** The arclength is:

$$
\int_0^2 \sqrt{(2t)^2 + (3t^2)^2} \, dt = \int_0^2 t \sqrt{4 + 9t^2} \, dt
$$

Now let $u = 4 + 9t^2$, so $du/18 = t \, dt$, and get

$$
\frac{1}{18} \int_4^{40} \sqrt{u} \, du = \frac{1}{18} \cdot \frac{2}{3} u^{3/2} \bigg|_4^{40} = \frac{1}{27} (40^{3/2} - 4^{3/2}) = \frac{8}{27} (\sqrt{1000} - 1) \approx 9.07.
$$

2. The parameterized curve $(3 \sin t, \cos t)$ gives an ellipse, with $t$ running from 0 to $2\pi$. Sketch the curve and then find the circumference of the ellipse.

**Solution:** The arclength is:

$$
\int_0^{2\pi} \sqrt{(3 \cos t)^2 + (-\sin t)^2} \, dt = \int_0^{2\pi} \sqrt{9 \cos^2 t + \sin^2 t} \, dt
$$

The integral is not elementary (it’s called an **elliptic integral**) and evaluates numerically to about 13.36.