1. Find the limit of each sequence, if it exists.

(a) \( a_n = \frac{n^2 + 1}{2n^2 + 1} \)

Solution: 0.5

(b) \( b_n = \cos \left( \frac{1}{n} \right) \)

Solution: 1

(c) \( c_n = \cos \left( \frac{\pi n}{2} \right) \)

Solution: Limit does not exist.

(d) \( d_n = \frac{n!}{4^n} \)

Solution: Limit does not exist (it diverges to \( \infty \)).

(e) \( e_1 = 1 \) and \( e_n = \cos(e_{n-1}) \)

Solution: \( \approx 0.7390851 \ldots \)

(f) \( f_n = n \sin \left( \frac{1}{n} \right) \)

Solution: 1

(g) \( g_n = \frac{1}{\sin(n)} \)

Solution: Limit does not exist.

(h) \( h_1 = 2 \) and \( h_{n+1} = \frac{h_n^2 + 2}{2h_n} \)

Solution: \( \sqrt{2} \)

(i) \( k_n = n! \sin(\pi n) \)

Solution: 0

(j) \( \gamma_n = \int_1^n \frac{1}{x} \, dx - \left( \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} \right) \)

Solution: This limit, known as the Euler-Mascheroni constant is approximately 0.57721.