1. Does the series converge? Explain why or why not.

(a) \[ \sum_{n=0}^{\infty} \frac{2}{n^2 + 1} \]

**Solution:** This converges by comparison with \[ \sum_{n=0}^{\infty} \frac{1}{n^2} \], since \[ \frac{2}{n^2 + 1} < \frac{2}{n^2} \] and \[ \sum_{n=0}^{\infty} \frac{1}{n^2} \] is a \( p \)-series with \( p > 1 \). You can also see this by the integral test, since \[ \int_{0}^{\infty} \frac{2}{x^2 + 1} \, dx = 2 \arctan(x) \bigg|_{0}^{\infty} = \pi < \infty \].

(b) \[ \sum_{n=1}^{\infty} \frac{1}{2n} \]

**Solution:** This diverges since \[ \sum_{n=0}^{\infty} \frac{1}{2n} = \frac{1}{2} \sum_{n=0}^{\infty} \frac{1}{n} \], and the harmonic series diverges.

(c) \[ \sum_{n=2}^{\infty} \frac{1}{n^2 \log(n)} \]

**Solution:** This converges. Compare with \[ \sum_{n=2}^{\infty} \frac{1}{n^2} \], since \[ \frac{1}{n^2 \log(n)} < \frac{1}{n^2} \].

(d) \[ \sum_{n=0}^{\infty} e^n + n \]

**Solution:** This diverges since the terms \[ \frac{e^n + n}{n^2 + 2n} \to 1 \] (not zero!) as \( n \to \infty \).

(e) \[ \sum_{n=1}^{\infty} \sqrt{\frac{n^3 - 1}{n^6 + 1}} \]

**Solution:** This converges. \[ \sqrt{\frac{n^3 - 1}{n^6 + 1}} < \sqrt{\frac{n^3}{n^6}} = \frac{1}{n^{3/2}} \]. The series converges by comparison with the convergent series \[ \sum_{n=1}^{\infty} \frac{1}{n^{3/2}} \].

(f) \[ \sum_{n=1}^{\infty} \frac{1}{n^2 - 5n/2} \]

**Solution:** This converges. First, you want to ignore the \( n = 1 \) and \( n = 2 \) term since they are negative. Then, use the integral test:

\[ \int_{3}^{\infty} \frac{dx}{x^2 - 5x/2} = \left[ \frac{2}{5} \log \left( \frac{x - 5/2}{x} \right) \right]_{3}^{\infty} = \frac{2}{5} \log 6 < \infty. \]

Alternately, you can use the comparison test. You’d like to compare with \[ \sum_{n=1}^{\infty} \frac{1}{n^2} \] but the terms of our series are larger than \[ \frac{1}{n^2} \]. The trick is to notice that eventually \( 5n/2 < n^2/2 \). This happens for all \( n > 5 \). Then \( n^2 - 5n/2 > n^2 - n^2/2 = n^2/2 \) so that \[ \frac{1}{n^2 - 5n/2} < \frac{2}{n^2} \]. The series converges since \[ \sum_{n=1}^{\infty} \frac{2}{n^2} \] converges.