Math 1520 – Quiz 3 – Take Home

This quiz should take you approximately 25 minutes. You may use your book, calculators, software, the internet, and any reference material, but do not work together and do not get help (except from Dr. Clair). Show work.

(10) 1. Let \( F(x) = \int_{1}^{x} \sqrt{1 - \ln(t)} \, dt. \)

(a) Find \( F(1). \)

(b) Find \( F'(1). \)

(c) Find \( F''(1). \)

**Solution:** a. 0; b. 1; c. \(-1/2\)

(10) 2. Evaluate the indefinite integral \( \int (2x + 5) \cos(x^2 + 5x + 4) \, dx. \)

**Solution:**

\[
\int (2x + 5) \cos(x^2 + 5x + 4) \, dx = \sin(x^2 + 5x + 4) + C
\]
3. Evaluate the indefinite integral $\int x e^{3x-2} \, dx$.

**Solution:**
\[
\int x e^{3x-2} \, dx = \frac{1}{3} x e^{3x-2} - \frac{1}{9} e^{3x-2} + C
\]

4. (a) Give an example of a rational function whose denominator is quadratic with two distinct roots.

(b) Find an antiderivative to the function you gave in part (a).

**Solution:**
(a) For example, $f(x) = \frac{1}{x(x+1)}$.

(b)
\[
\int f(x) \, dx = \int \frac{1}{x} - \frac{1}{x+1} \, dx = \log(x) - \log(x+1) + C
\]
(10) 5. The graph of \( f(x) \) is shown below. Estimate \( \int_{2}^{3} x f(x^2) \, dx \).

**Solution:** Use the substitution \( u = x^2 \), so \( du = 2x \, dx \), and

\[
\int_{2}^{3} x f(x^2) \, dx = \frac{1}{2} \int_{4}^{9} f(u) \, du.
\]

Now estimate the area under the curve \( f \) from 4 to 9, which is almost exactly a triangle, by \( \frac{1}{2}(5 \cdot 5) = \frac{25}{2} \), so the integral is approximately \( \frac{25}{4} = 6.25 \).