MATH 266-01 (091) TEST 1

1. (10 pts) Suppose that $P$, $Q$, and $R$ are propositions such that $P$ is true, $Q$ is true, while $R$ is false. Determine whether

$$(P \lor R) \implies (Q \land R)$$

is true or false. Justify your answer completely (a “partial truth table” would be sufficient).

2. (10 pts) Give a useful denial of the statement: “Either $f(x)$ is not continuous at $x = 0$, or $f(x)$ is differentiable at $x = 0$”.

3. (20 pts) (a) Identify the antecedent and the consequent: “13 is a prime number is necessary for $3 > 6$.”

(b) Write the converse and contrapositive of the statement in part (a).

Converse:

Contrapositive:
3. (c) Is the statement in 3(a) true or false? Explain.

4. (24 pts) Translate the sentence into a symbolic sentence with quantifiers. The universe is given in parentheses.

(a) All zebras have stripes. (animals)

(b) No one loves everybody. (people)

(c) For every real number $x$, there is a non-negative real number $y$ such that $y = x^2$. (real numbers)
5. (12 pts) Let $x$ be an integer. Give a direct, formal proof of the statement: if $x$ is even, then $x^2$ is divisible by 4.

6. (12 pts) Use the technique of working backward from the desired conclusion to prove that for positive numbers $x$ and $y$,

$$\sqrt{x^2 + y^2} < x + y.$$
7. (12 pts) Let $a$ and $b$ be positive integers. Use the method of contraposition to prove that if $x + y$ is odd, then either $a$ is odd or $b$ is odd.