1. Suppose that the plane is colored with three colors (red, green and blue.) Is it possible to
color the plane so that every two points that are a distance 1 from each other have different
colors?

2. Let $f(x)$ be a polynomial with positive integer coefficients. (a) Prove that $f(1)$ divides
$f(f(1)+1)$.

(b) Prove that if $n \geq 2$, then $f(n)$ does not divide $f(f(n)+1)$.

3. Let $k$ be a natural number and let $C$ be a positive number. Prove that there is a natural
number $N$ such that for all $n \geq N$, $Cn^k < 2^n$ holds.

4. Use 3. to show that for any polynomial $f(x)$ there is a natural number $N$ such that for all
$n \geq N$, $f(n) < 2^n$ holds.

5. Show that every positive integer is a sum of one or more integers of the form $2^j3^k$, where
$j$ and $k$ are non-negative integers and where no summand divides another summand.