1. Referring to the game of SubDivvy. Recall that a player has a *winning strategy* if, no matter how her opponent moves, she is able to make moves so as to win the game. In other words, a player has a winning strategy if he can force a win. Recall that a whole number $N$ is a *winning position* if a player with position $N$ has a winning strategy.

(a) Write a short paragraph that explains why $N = 4$ and $N = 6$ are winning positions.

(b) Formulate a definition of “$N$ is a losing position.” Give examples (values of $N$) for your definition, and explain why they are examples.

(c) Formulate a conjecture about which numbers are winning positions and which numbers are losing positions. Are there numbers that are neither winning nor losing positions?

2. For each of the following conditions, give *explicit* examples of two non-empty sets $A$, one where $|A|$ is finite, and another where $|A|$ is infinite.

(a) $A \subset \mathcal{P}(\mathbb{R})$,

(b) $A \in \mathcal{P}(\mathbb{R})$.

3. Let $A$ and $B$ be subsets of some universal set $U$, and consider the equation

$$\widehat{A \cup B} = \widehat{A} \cup \widehat{B}$$

(In the books notation for complements this would be $\overline{A \cup B} = \overline{A} \cup \overline{B}$, but I really don’t like the book’s notation for complements :) ) This equation is not true for all sets $A$ and $B$. Give a *specific* counterexample. Then find a similar equation that is true for all sets $A$ and $B$. Justify with a Venn diagram.

4. Informally discuss the relation between the sets $\mathcal{P}(A \cup B)$ and $\mathcal{P}(A) \cup \mathcal{P}(B)$.

5. Find an explicit indexed collection of sets $\{A_n\}_{n \in \mathbb{N}}$ that satisfies all of the following conditions:

(i) $A_1 \supset A_2 \supset A_3 \supset \cdots \supset A_n \supset A_{n+1} \supset \cdots$,

(ii) $\bigcup_{n=1}^{\infty} A_n = (-2, 2)$,

(iii) $\bigcap_{n=1}^{\infty} A_n = [-1, 1]$. 