Finish up 1.2

Mixing problem:

Have 10 gal. vat of water and dump salt in
at rate of ½ lb. per minute. Open spigot so that
¼ gal. leaves per min. Add pure water at
rate of ¼ gal/min. How much salt in the sol.
at time t? salt per gal.

\[
\frac{ds}{dt} = \frac{1}{2} - \frac{s}{10} \cdot \frac{1}{4}
\]

¼ gal leaves \[S = \text{amt of salt}\]

\[
\frac{ds}{dt} = \frac{1}{20-s} - \frac{s}{40}
\]

\[
\int \frac{ds}{20-s} = \frac{dt}{40} \quad -\ln(20-s) = \frac{t}{40} + C
\]

\[
\ln(20-s) = -\frac{t}{40} + C'
\]

\[
20-s = Ke^{-t/40} \quad s = 20 - Ke^{-t/40}
\]

\[
S(0) = 0 = 20 - Ke^{-0/40} \quad K = 20
\]

\[
S(t) = 20 - 20e^{-t/40}
\]

Do #33, #35 1.2

Read 1.3, 1.4

HW 1.3: 1, 5, 9, 11-23 odd

Slope field for diff eq

\[
\frac{dy}{dt} = f(y, t)
\]

At each pt. y, t can compute f(y, t) and draw line segment at (y, t) w/ slope f(y, t). That slope is dy/dt = slope of solution curve y(t) at (y, t).

Connect slopes to get an idea of
\[ \frac{dy}{dt} = t \]

Slope field

d one sol. curve

If \( \frac{dy}{dt} = f(t) \), what can you say about the slope field?

(If it is the same as move vertically)

About the solutions through \( t_0 \) and through \( t_0, y_s \)?

One is the vertical translate of the other.

The above is an example of this.

\( \text{diff eq is "autonomous"} \)

If \( \frac{dy}{dt} = g(y) \)?

\[ \frac{dy}{dt} = y \]

slope field is horizontally the same

Have them do \#2, \#4 by hand and then using HPC-Solver

\#12 \hspace{1cm} \frac{dS}{dt} = S^3 - 2S^2 + S = S(S^2 - 2S + 1) = S(S-1)^2

\[
\begin{align*}
\text{if } S &< 0 & S < 0 \\
\text{if } S &> 0 & S > 0 \\
\text{if } S &= 0 & S = 0
\end{align*}
\]
$\frac{RC}{dt} \frac{dv_c}{dt} + v_c = V(t)$

\[ RC \frac{dv_c}{dt} = -v_c \]

\[ \int \frac{dv_c}{v_c} = \int \frac{-dt}{RC} \]

\[ \ln|v_c| = -\frac{t}{RC} + k \]

\[ V_c(t) = V_0 e^{-\frac{t}{RC}} \]

\[ V_c(t) = V_0 e^{-\frac{t}{RC}} \]

Graph using $R = 0.2$, $C = 1$, $V_0 = 3$

\[ V(t) = \begin{cases} k & 0 \leq t < 3 \\ 0 & t > 3 \end{cases} \]

\[ \frac{dv_c}{dt} = \frac{k - v_c}{RC} \]

\[ \int \frac{dv_c}{v_c - k} = \int \frac{dt}{RC} \]

\[ \ln|v_c - k| = -\frac{t}{RC} + L \]

\[ V_c(t) = L e^{-\frac{t}{RC}} + K \]

\[ V_c(t) = (1-k)e^{-\frac{t}{RC}} + K = Pe^{-\frac{t}{RC}} \]

\[ P = (1-k) + Ke^{3RC} \]

\[ V_c(t) = [ (1-k) + Ke^{3RC} ] e^{-\frac{t}{RC}} \]

Discuss it gets shifted up
Euler's method: numerical approximation to solutions $(y_0, t_0)$ of a solution curve $f(t)$ at $t$, on that curve.

$$\frac{y_1 - y_0}{t_1 - t_0} = \frac{dy}{dt} \approx \frac{dy}{dt} = f(t_0, y_0)$$

$$y_1 - y_0 \approx f(t_0, y_0) [t_1 - t_0]$$

$$y_1 = f(t_0, y_0) [t_1 - t_0] + y_0 = f(t_0, y_0) \Delta t + y_0$$

**SO:** Start off at point $(t_0, y_0)$, then to approximate solution $y(t)$ with initial cond. $y(t_0) = y_0$ go in steps of $\Delta t$ from $t_0$ to $t_0$, using the above approximation.

$$t_1 = t_0 + \Delta t \quad y_1 = f(t_0, y_0) \Delta t + y_0$$

$$t_2 = t_1 + \Delta t \quad y_2 = y(t_2) \approx f(t_1, y_1) \Delta t + y_1$$

$$\text{eq: } \frac{dy}{dt} = yt$$

$(t_0, y_0) = (0, 1) \quad \Delta t = .2$  

Approximation $y(1)$

$t_1 = 0.2 \quad y_1 = f(0.1) \cdot 0.2 + 1 = 0.1 + 1 = 1$

$t_2 = 0.4 \quad y_2 = f(0.2) \cdot 0.2 + 1 = 0.2 \cdot 0.2 + 1 = 1.04$

$t_3 = 0.6 \quad y_3 = f(0.4) \cdot 0.2 + y_2 = 0.4 \cdot 1.04 \cdot 0.2 + 1.04 = 0.83 + 1.04 = 1.832$

$t_4 = 0.8 \quad y_4 = f(0.6) \cdot 0.2 + y_3 = 0.6 \cdot 1.232 \cdot 0.2 + 1.1232 = 1.257984$

$t_5 = 1 \quad y_5 = 0.8 \cdot 1.257984 \cdot 0.2 + 1.257984 = 1.45926144 \approx y(1)$

**Actual solution:** $\frac{dy}{dt} = t \ dt$  

check $y = ke^{\frac{t^2}{2}}$  

$y(0) = 1 \ ; \ y_0 = e^{\frac{0^2}{2}} = 1$  

$\Rightarrow y(1) = 1.45926144$