Another example: \[ \frac{dy}{dt} = f_x(y) = y^3 + \alpha \]

Phase lines:

- \( \alpha = 1 \)
- \( \alpha = 0 \)
- \( \alpha = -1 \)

\[ \text{all sources have a source at } y = -\frac{1}{\sqrt[3]{2}} \text{ for all } \alpha. \text{ No other equal pt.} \]

\( \rightarrow \text{No Bifurcation} \)

What is the difference?

Suppose partial derivatives of \( f(y, \alpha) = f_x(y) \) w.r.t. \( y \) and \( \alpha \) exist and are continuous. Then changes in \( \alpha \) result in a change in \( f_x(y) \).

\[ f'_x(y^*) \text{ both have } f''_x(y^*) > 0 \text{ so both are sources} \]

Both equilibrium pts are sources \[ \boxed{\text{NO BIFURCATION}} \]

Look for an \( \alpha \) which has an equilibrium pt \( y^* \) where \( f''_x(y^*) = 0 \). This is where may have BIFURCATION VALUE.

\[ \text{Eqn.} \frac{dy}{dt} = f_x(y) = y^3 + \alpha y^2 = y^2(y + \alpha) \]

Equal pts at \( y = 0 \), \( y = -\alpha \)

\[ f'_x(y) = 3y^2 + 2\alpha y \rightarrow \begin{cases} \alpha = 0 & f'_x(0) = 0 \end{cases} \]

\[ f''_x(y) = 6y + 2\alpha > 0 \text{ at } y > 0 \]

\[ \Rightarrow \alpha \neq 0, f''_x(-\alpha) > 0 \]

Only possible bifurcation \( \alpha = 0 \),

[Diagram of phase lines and trajectories]
Have them draw sketches

\[ f(y) + 2 \]

\[ f(y) + \alpha \]

\[ f(y) - 2 \]

\[ \frac{dP}{dt} = kP(1 - \frac{P}{N}) - c^{\text{harvesting}} \]

Equil sol:

\[ p = \frac{N}{2} \pm \sqrt{\frac{N^2}{4} - \frac{CN}{K}} \]

\[ \frac{N^2}{4} - \frac{CN}{K} > 0 \rightarrow 2 \text{ eq pts (} c < \frac{KN}{4} \text{)} \]

Equil sol:

\[ c = 0 \]

\[ c = \frac{KN}{4} \]

\[ c \rightarrow \text{window gets smaller} \]