MT-A143-01 Exam Three Fall 2004

You may keep this page of questions. Turn in your answers with all of your work on the yellow paper and the green paper. You are NOT allowed to use your calculator on the first five questions. Answer these five questions on the yellow paper. When you have completed these five questions, turn in all of the yellow paper and receive green paper to use on the last two questions. You ARE allowed to use your calculator on the last two questions and you will need your calculator for some parts of the last two questions.

I. (1) 12 Points. Find the Maclaurin series for \( f(x) = 1 - e^{-3x} \). You are expected to use a known power series and to express your final answer using summation notation.

(2) 12 Points. Find the degree 2 Taylor polynomial for \( f(x) = \sqrt{4x + 1} \) about \( a = 2 \).

(3) 16 Points. Find the interval of convergence, including endpoint behavior, for the power series
\[
\sum_{k=1}^{\infty} \frac{(x - 6)^k}{(k^2 + 1)5^k}.
\]

(4) 20 Points. Let \( f(x) = \begin{cases} k(\sqrt{x})(1 - x) & \text{if } 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \)
(a) Find the value of \( k \) required for \( f(x) \) to be a probability density function.
(b) Find the mean, \( \mu \), for this probability distribution.

(5) 10 Points. Find an exact value for the sum of the following series by recognizing the series as a Maclaurin series evaluated at a particular value of \( x \).
\[
1 - \frac{(\pi/6)^2}{2!} + \frac{(\pi/6)^4}{4!} - \frac{(\pi/6)^6}{6!} + \frac{(\pi/6)^8}{8!} - \frac{(\pi/6)^{10}}{10!} + \cdots
\]

(6) 10 Points. If you save $500.00 at the beginning of every year for 40 years and invest this money at 5.2% nominal annual interest compounded annually, how much money will you have at the end of 40 years?

(7) (a) 6 Points. Find the partial sums \( S_{100}, S_{200} \) and \( S_{400} \) for the series
\[
\sum_{k=1}^{\infty} \frac{1}{(2k - 1)^{1.25}}
\]
(b) 10 Points. Use the integral test to prove that this series converges. Show your work!
(c) 4 Points. Approximate the sum of this series to the nearest thousandth. [Hint: Use the approximation part of the integral test.]