You may keep this page of questions. Turn in your answers with all of your work on the colored paper. You may use your calculator ONLY on questions 6 and 7.

16 Points. Match each of the following differential equations with one of the slopefields below.

(a) \( \frac{dy}{dx} = 2y^2 + 3y \)  \quad (b) \( \frac{dy}{dx} = x^2 - y^2 \)  \quad (c) \( \frac{dy}{dx} = -xy \)  \quad (d) \( \frac{dy}{dx} = \frac{x}{\sqrt{y^2 + 1}} \)
(2) 12 Points. The slope field below is a slope field for the differential equation \( \frac{dy}{dx} = \cos(x^2 - 2y) \). On this slope field sketch a graph of the solution of the initial value problem

\[
\frac{dy}{dx} = \cos(x^2 - 2y), \quad y(-2.0) = -1.0.
\]
(2) 12 Points. On the yellow page, I have printed for you a slope field for the differential equation \( \frac{dy}{dx} = \cos(x^2 - 2y) \). On this slope field sketch a graph of the solution of the initial value problem

\[
\frac{dy}{dx} = \cos(x^2 - 2y), \quad y(-2.0) = -1.0.
\]

(3) 12 Points. Which, if any, of the following functions are solutions of the differential equation \( 3x^2 \frac{d^2y}{dx^2} - 2x \frac{dy}{dx} - 2y = 0 \)?

(a) \( y = e^x \)  
(b) \( y = x^2 \)  
(c) \( y = x^3 \)  
(d) \( y = \frac{1}{3}x \).

(4) (a) 6 Points. Find all equilibrium solutions of the differential equation

\[
\frac{dy}{dx} = 0.1(y + 2)(y - 1)(y - 5).
\]

(b) 8 Points. Sketch a slope field for the differential equation in (a) and use it to determine whether each equilibrium solution is stable or unstable.

(5) 18 Points. Solve the initial value problem \( \frac{dy}{dx} = xe^{-2y}, \quad y(0) = 3 \)

(6) 12 Points. Use Euler’s method with step size \( h = 0.5 \) to find the values \( y(1.5), y(2.0) \) and \( y(2.5) \) for the Euler approximation to the solution of the initial value problem

\[
\frac{dy}{dx} = (2x)^2 + y, \quad y(1.0) = 2.0.
\]

(7) 16 Points. Water leaks out of the bottom of a barrel at a rate which is directly proportional to the square root of the depth of the water in the barrel at that time. If the water level starts at 64.0 centimeters and drops to 60.0 centimeters after 4.00 hours, how long will it take for all of the water to leak out of the barrel? Give both an exact answer, in hours, and a decimal approximation to the nearest minute.

[The answer 64 hours is WRONG. For partial credit, model this question with a differential equation and solve the differential equation.]