I. Analyze and evaluate the following definite, indefinite, or improper integrals.

(1) \( \int_0^{\pi/6} \sin(x) \cos(x) \, dx \)  
(2) \( \int t^3 \sin(2t) \, dt \)  
(3) \( \int \sqrt{4-x^2} \, dx \)  
(4) \( \int_2^\infty \frac{dx}{(x+2)^2} \)  
(5) \( \int \frac{dt}{t^2 + 9t + 10} \)  
(6) \( \int \frac{5x^2 - 12x - 20}{x^3 - 4x} \, dx \)  
(7) \( \int \sinh^{-1}(bw) \, dw \) where \( b \neq 0 \).

(8) Find the Maclaurin series for \( y = f(x) = xe^{-3x} \). Express your final answer using summation notation.

(9) Solve the initial value problem: \( \frac{dy}{dx} = 3e^{-y}x^2, \quad y(0) = 4 \).

(10) Solve the initial value problem:

\[
y'' - 3y' - 10y = 0, \quad y(0) = 5, \quad y'(0) = -3.
\]

(11) 16 Points. Find the interval of convergence for the following power series. At the endpoints of the interval, either prove convergence of the series or else prove divergence.

\[
\sum_{k=0}^{\infty} \frac{(-1)^k(x + 5)^k}{(k^2 + 1)2^k}.
\]
(12) 14 Points. Find the volume of the solid of revolution that is generated by revolving the region bounded by \( x = 3, \ x = 5, \ y = 0, \) and \( y = \frac{1}{x^2} \) about the \( y \)-axis.

(13) 14 Points. Find the mass of the region in the \( xy \)-plane that is bounded by \( y = x \) and \( x = y^2 - 3y \) if the density \( \delta \) at a point \((x, y)\) within the region is given by \( \delta = \sqrt{y} \).

(14) 10 Points. Let \( f(x) = \begin{cases} 0 & \text{if } x < 0 \\ kxe^{-x^2} & \text{if } x \geq 0 \end{cases} \) for what value of \( k \) will \( f(x) \) be a probability density function?

(15) 12 Points. Using the table for \( f(x) \) below, find the numerical approximations \( T_6 \) and \( S_6 \) for the integral \( \int_1^4 f(x) \, dx \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>1.0</th>
<th>1.5</th>
<th>2.0</th>
<th>2.5</th>
<th>3.0</th>
<th>3.5</th>
<th>4.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>5.17</td>
<td>4.83</td>
<td>4.92</td>
<td>5.58</td>
<td>5.93</td>
<td>6.22</td>
<td>6.41</td>
</tr>
</tbody>
</table>

(16) 14 Points. A demographer studying the population of a certain small country uses the logistic model

\[ P = \frac{L}{1 + Ae^{-kt}} \]

The population of the country was 2.403 million at the beginning of 1970. From a careful analysis of annual population studies, the demographer estimates that the inflection point for the logistic curve occurred at the beginning of 2004 when the population was 3.145 million. Evaluate the parameters for the logistic model. Using these, what population does the logistic model predict at the beginning of 2030?