MT-A311-01 Final Exam Spring 2000

You may keep this page of questions. Turn in your answers with all of your work on the colored paper. You are NOT allowed to use calculators or Mathcad on this exam.

Questions # 1–5 are worth 20 points each. Questions # 6–9 are worth 25 points each.

(1) State a definition for each of the following:
(a) The two \( n \times n \) matrices \( A \) and \( B \) are similar.
(b) \( A \) is a nonsingular \( n \times n \) matrix.
(c) The matrix \( A \) is Hermitian.
(d) The set \( \{v_1, v_2, \ldots, v_n\} \) of vectors is a linearly independent set.
(e) The matrix \( P \) is an orthogonal matrix.

(2) Use row operations to find \( A^{-1} \) if
\[
A = \begin{bmatrix}
1 & 1 & 0 & -1 \\
2 & 1 & 1 & 2 \\
0 & 1 & 1 & -1 \\
1 & 0 & -1 & 2
\end{bmatrix}.
\]

(3) Given that \( \mathcal{B} = \left\{ \begin{bmatrix} 2 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} \right\} \) is a linearly independent set and that
\[
v = \begin{bmatrix} 1 \\ -7 \\ 6 \end{bmatrix} \in \text{Span}(\mathcal{B}),
\]
find \( v_B \).

(4) Find determinants for the following four matrices. None of these four should require really lengthy calculations.
\[
A = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 \\
3 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -3 & 0 \\
0 & 0 & 0 & 0 & 0 & 4
\end{bmatrix}
\]
\[
B = \begin{bmatrix}
2 & 0 & 0 & 0 \\
0 & -3 & 0 & 0 \\
0 & 0 & 5 & 0 \\
0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & \pi
\end{bmatrix}
\]
\[
C = \begin{bmatrix}
2 + i & 3i \\
4 & 2 - i
\end{bmatrix}
\]
\[
D = \begin{bmatrix}
1 & 1 & 1 \\
2 & 3 & 4 \\
2 & 9 & 16 \\
8 & 27 & 64 \\
4 & 5 & 25
\end{bmatrix}
\]
(5) Find the Wronskian for the set of functions \( \{e^{2x}, xe^{2x}, x^2e^{2x}\} \). Simplify!

(6) (a) Use Gauss-Jordan elimination to solve the system of equations:

\[
\begin{align*}
    x_1 - 2x_2 - x_3 + x_4 + 3x_5 &= -4 \\
    2x_1 - 4x_2 - x_3 + 5x_4 + 4x_5 &= -3 \\
    x_1 - 2x_2 + 2x_3 + 10x_4 - 3x_5 &= 11 \\
    3x_1 - 6x_2 - 3x_3 + 3x_4 + 8x_5 &= -10
\end{align*}
\]

(b) Find the rank and the nullity for the matrix

\[
A = \begin{bmatrix}
1 & -2 & -1 & 1 & 3 \\
2 & -4 & -1 & 5 & 4 \\
1 & -2 & 2 & 10 & -3 \\
3 & -6 & -3 & 3 & 8
\end{bmatrix}
\]

which is the matrix of coefficients for the system of equations in (6)(a).

(7) Find the eigenvalues and eigenvectors for the matrix

\[
A = \begin{bmatrix}
0 & 2 & -10 \\
1 & 0 & 1 \\
0 & 1 & 4
\end{bmatrix}
\]

given that the characteristic polynomial for \( A \) is \( P(\lambda) = (\lambda - 3)^2(\lambda + 2) \).

(8) Use the simplex method to solve the following linear programming problem.

Maximize \( z = 24x_1 + 10x_2 + 16x_3 \), subject to \( x_i \geq 0 \), \( i = 1, 2, 3 \), and

\[
\begin{align*}
6x_1 + 2x_2 + 5x_3 &\leq 300 \\
8x_1 + 4x_2 + 5x_3 &\leq 480
\end{align*}
\]

(9) Express the matrix

\[
B = \begin{bmatrix}
1 & 1 & 1 \\
-1 & 1 & 2 \\
1 & 1 & 3
\end{bmatrix}
\]

as a product \( QR \) where \( Q \) is orthogonal and \( R \) is upper triangular.