(1) 10 Points. If the reduced row echelon form for the matrix \([A|b]\) is the matrix

\[
R = \begin{bmatrix}
1 & -4 & 0 & 2 & 0 & -5 & 4 \\
0 & 0 & 1 & -3 & 0 & 7 & 6 \\
0 & 0 & 0 & 0 & 1 & 3 & -9 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

find the general solution of the system of equations \(Ax = b\).

(2) 8 Points. What are the dimensions for the four subspaces of the matrix \(B\) if \(B = \begin{bmatrix} I & O \\ I & O \end{bmatrix}\) where \(I\) is the 4 by 4 identity matrix and \(O\) is the 4 by 3 zero matrix?

(3) 12 Points. Which of the following subsets of \(\mathbb{R}^3\) are actually subspaces.
   (a) the set of all vectors \((x, y, y)\) such that \(x\) and \(y\) are real numbers,
   (b) the set of all vectors \((x, 1, z)\) such that \(x\) and \(z\) are real numbers,
   (c) the set of all vectors \((x, y, z)\) such that \(x - y + 4z = 0\),
   (d) the set of all vectors \((x, y, z)\) such that \(xyz = 0\).

(4) 8 Points. If \(S\) is the subspace of \(\mathbb{R}^4\) spanned by \((3, 0, 0, 0), (0, 0, -2, 0)\) and \((0, 0, 0, 5)\), what is \(S^\perp\)?

(5) 12 Points. For which value or values of \(b\) is the set

\[
\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ b \\ 3 \end{bmatrix} \right\}
\]

a linearly independent set of vectors?
(6) 30 Points.

Let

\[ A = \begin{bmatrix}
1 & 1 & 1 & 1 & 1 \\
1 & 2 & 1 & 1 & 1 \\
2 & 3 & 2 & 2 & 2 \\
1 & 5 & 1 & 1 & 1
\end{bmatrix} \]

(a) Find the reduced row echelon form for \( A \).
(b) Find the reduced row echelon form for \( A^T \).
(c) Find a basis for the row space of \( A \).
(d) Find a basis for the nullspace of \( A \).
(e) Find a basis for the column space of \( A \).
(f) Find a basis for the left nullspace of \( A \).

(7) 20 Points. If \( V \) is the subspace of \( \mathbb{R}^4 \) that is spanned by

\[ \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -1 \\ 0 \end{bmatrix} \right\} \quad \text{and} \quad b = \begin{bmatrix} 10 \\ 1 \\ 0 \\ 2 \end{bmatrix}, \]

find vectors \( p = \overrightarrow{\text{proj}_V}(b) \in V \) and \( n = \overrightarrow{\text{proj}_{V^\perp}}(b) \in V^\perp \) such that \( b = p + n \).