MT-A403-01 Exam Two Spring 2000

You may keep this list of questions. Turn in all of your work with your answers on the colored paper.

(1) 12 Points. Specifications for a certain job call for washers with an inside diameter that is no less than 0.244 inches and no more than 0.256 inches. If the inside diameters of the washers supplied by a given manufacturer may be looked upon as a random variable having the normal distribution with \( \mu = 0.251 \) inch and \( \sigma = 0.004 \) inch, what percentage of these washers will meet the specifications?

(2) 18 Points.
(a) If \( Z \) has a standard normal distribution, find \( z_{0.03} \).
(b) Find \( \chi^2_{0.25} \) for a Chi-Square distribution with 17 degrees of freedom.
(c) Find \( f_{0.01} \) for an \( F \)-distribution with \( \nu_1 = 4 \) and \( \nu_2 = 13 \).

(3) 24 Points. The random variable \( X \) has the following density function:

\[
f(x) = \begin{cases} 
  kx^{-3} & \text{if } 1 \leq x \leq 3 \\
  0 & \text{elsewhere}
\end{cases}
\]

(a) Find the value of \( k \).
(b) Find \( E(X) \).
(c) Find \( \text{Var}(X) \).

(4) 10 Points. The amount of time that a surveillance camera will run without having to be reset is a random variable having the exponential distribution with a mean of 36 days. Find the probability that such a camera will have to be reset in less than 30 days.

(5) 24 Points. Let \( R \) be the region in the \( xy \)-plane that is bounded by \( x = 0, y = 1, \) and \( y = x \). Let \( X \) and \( Y \) have the joint density function given by

\[
f(x, y) = \begin{cases} 
  24x(1 - y) & \text{if } (x, y) \in R \\
  0 & \text{elsewhere}
\end{cases}
\]

(a) Find the marginal density function \( f_2(y) \) for \( Y \).
(b) Find \( P(X \geq \frac{1}{2}) \).

(6) 12 Points.
(a) State Chebyshev’s Theorem.
(b) Write a brief essay discussing both the usefulness of Chebyshev’s Theorem and its limitations.