(1) 20 Points. A random sample of size 1600 is taken from an infinite population having population mean $\mu = 512.00$ and population variance $\sigma^2 = 100.00$. Find the probability that a sample mean from this population will have a value that is between 511.93 and 512.36.

(2) 16 Points. Suppose that $X$ and $Y$ are independent random variables with $E(X) = -3$, $E(Y) = 4$, $\text{Var}(X) = 16$, and $\text{Var}(Y) = 25$. Find the expected value of $W$ and the variance of $W$ if $W = 5X - 3Y + 11$.

(3) 20 Points. During one stage in the manufacture of integrated circuit chips, a coating must be applied. If 75% of chips receive a thick enough coating, find the probability that among 18 chips at least 16 will have thick enough coatings.

(4) 16 Points. In a small geology class, each of five students must write a report on one of nine field trips. In how many different ways can they each choose one of the field trips if no two students may choose the same field trip?

(5) 20 Points. Find an 80% confidence interval for the population variance of a normal population if a random sample of size 12 taken from the population had a sample variance $s^2 = 43.40$.

(6) 30 Points. The random variable $X$ has the following density function:

$$f(x) = \begin{cases} \frac{4}{27}x^2(3 - x) & \text{if } 0 \leq x \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Find $E(X)$.
(b) Find $\text{Var}(X)$.
(c) Find $P(X \leq 2)$. 
(7) 40 Points. The following data were obtained for a randomly chosen sample of size \( n = 5 \) from a bivariate normal population.

<table>
<thead>
<tr>
<th>x</th>
<th>9</th>
<th>3</th>
<th>6</th>
<th>5</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>4</td>
<td>10</td>
<td>10</td>
<td>14</td>
<td>17</td>
</tr>
</tbody>
</table>

(a) Draw a scatterdiagram for this set of data.

(b) Calculate the following sums. **Check your work!** Wrong answers here will lead to wrong answers on parts (c), (d) and (e) below.

\[
\begin{align*}
\sum x_i &= \, \\
\sum y_i &= \\
\sum x_i^2 &= \\
\sum x_i y_i &= \\
\sum y_i^2 &= \\
\end{align*}
\]

(c) Find the equation for the estimated regression line.

(d) Find the sample correlation coefficient.

(e) We use this sample for the hypothesis test

\[
H_0 : \rho = 0 \\
H_1 : \rho \neq 0
\]

Should we accept or reject the null hypothesis at the 5% level of significance? Show your work!

(8) 20 Points. Suppose that colored balls are distributed in 3 boxes as follows:

- box 1 contains 2 red balls, 1 white ball and 2 blue balls
- box 2 contains 6 red balls, 5 white balls and 9 blue balls
- box 3 contains 7 red balls, 2 white balls and 1 blue ball

One of the boxes is selected by a procedure for which there is a 35% probability of selecting box 1, a 40% probability of selecting box 2, and a 25% probability of selecting box 3. From the selected box, a ball is selected at random. If the ball is red, what is the probability that it is from box 3?

(9) 18 Points. (a) What is required for a function \( f(x, y) \) to be a joint probability density function for the two continuous random variables \( X \) and \( Y \).

(b) Given a continuous joint density function \( f(x, y) \) as in part (a), state the definitions for the corresponding marginal density functions \( f_1(x) \) and \( f_2(y) \).

(c) When are the random variables \( X \) and \( Y \) independent of each other? **If** you choose to answer this question in terms of the distribution functions \( F_1(x), F_2(y) \) and \( F(x, y) \), then you **MUST** explain what these distribution functions are.