MT-P403 Exam Two Spring 1998

You may keep this list of questions. Turn in all of your work with your answers on the colored paper.

(1) 10 Points. If $X$ has a normal distribution with mean $\mu = 7.00$ and variance $\sigma^2 = 25.00$, find

\[ P(X > 13) \]
\[ P(-3.00 \leq X \leq 4.00) \]

(2) 20 Points.

(a) If $Z$ has a standard normal distribution, find $z_{0.12}$.
(b) Find $\chi^2_{0.01}$ for a Chi-Square distribution with 7 degrees of freedom.
(c) Find $f_{0.05}$ for an $F$-distribution with $\nu_1 = 8$ and $\nu_2 = 5$.
(d) Find $f_{0.99}$ for an $F$-distribution with $\nu_1 = 3$ and $\nu_2 = 10$.

(3) 18 Points. The random variable $X$ has the following density function:

\[ f(x) = \begin{cases} 
  k(4x - x^3) & \text{if } 0 \leq x \leq 2 \\
  0 & \text{elsewhere}
\end{cases} \]

(a) Find the value of $k$.
(b) Find $E(X)$.
(c) Find $\text{Var}(X)$.

(4) 10 Points. Assume that planet E is bombarded by asteroids and that the time $T$ between major asteroid collisions has an exponential distribution with a mean of 800 planet E years. Find the probability that a major asteroid collision will occur on planet E in the next 600 planet E years.
(5) 10 Points. Given that the density function $f(x; \alpha, \beta)$ for the Beta distribution is

$$
f(x; \alpha, \beta) = \begin{cases} 
\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1} & \text{if } 0 < x < 1 \\
0 & \text{elsewhere}
\end{cases}
$$

(a) Simplify $f(x; 6, 2)$.

(b) Find the distribution function $F(x; 6, 2)$.

(6) 20 Points. Let $R$ be the region in the $xy$-plane that is bounded by $y = 0, x = 3,$ and $x = 3y$. Let $X$ and $Y$ have the joint density function given by

$$
f(x, y) = \begin{cases} 
\frac{2}{7} (x + y) & \text{if } (x, y) \in R \\
0 & \text{elsewhere}
\end{cases}
$$

(a) Find the marginal density function $f_1(x)$ for $X$.

(b) Find $P(Y \leq \frac{1}{2})$.

(7) 12 Points.

(a) State Chebyshev’s Theorem.

(b) Write a brief essay discussing both the usefulness of Chebyshev’s Theorem and its limitations.