MT-P403 Make-Up Exam Two Spring 1998

You may keep this list of questions. Turn in all of your work with your answers on the colored paper.

(1) 20 Points.
(a) Find \( P(4.33 \leq X \leq 10.72) \) if \( X \) has a normal distribution with mean 8.50 and variance 9.00.
(b) If \( Z \) has a standard normal distribution, find \( z_{0.025} \).
(c) Find \( \chi^2_{0.025} \) for a Chi-Square distribution with 23 degrees of freedom.
(d) Find \( t_{0.90} \) for a \( T \)-distribution with 14 degrees of freedom.

(2) 30 Points. The random variable \( X \) has the following density function:
\[
f(x) = \begin{cases} 
    kx^2e^{-x} & \text{if } x > 0 \\
    0 & \text{elsewhere}
\end{cases}
\]
(a) Find the value of \( k \).
(b) Find \( E(X) \).
(c) Find the distribution function for \( X \).

(3) 10 Points. If \( S_1^2 \) and \( S_2^2 \) represent the variances of independent random samples of size \( n_1 = 7 \) and \( n_2 = 10 \) taken from normal populations with equal variances, find \( P(S_1^2/S_2^2 > 5.80) \).

(4) 24 Points. Let \( R \) be the region in the \( xy \)-plane that is bounded by \( y = 0, x = 0, \) and \( y = \sqrt{4-x} \). Let \( X \) and \( Y \) have the joint density function given by
\[
f(x, y) = \begin{cases} 
    \frac{3}{16}xy & \text{if } (x, y) \in R \\
    0 & \text{elsewhere}
\end{cases}
\]
(a) Find the marginal density function \( f_1(x) \) for \( X \).
(b) Find \( P(X \leq 3Y) \).

(5) 16 Points.
(a) Carefully state the definition for the independence of two random variables \( X \) and \( Y \).
(b) Write a brief essay explaining why the independence of random variables is an important concept.