The Physics of Bowling:
How good bowlers stay off the straight and narrow.

Brody Dylan Johnson

Saint Louis University
Outline

1. Introduction
2. Bowling Strikes
3. Underlying Physics
4. Mathematical Model
5. Human Experiment
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Lane dimensions: Length: 60 feet, Width: 41.5 inches.

Ball specifications: Circumference: 2.25 feet, Weight: up to 16 pounds.

Scoring

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- In the case of a spare (all remaining pins knocked down on second ball) the pinfall of the next ball is added to the score for the frame;
- These bonus pinfalls can lead to up to two additional frames when a spare or strike occurs in the tenth frame.
A perfect game consists of twelve consecutive strikes. The pinfall is 30 for each frame for a total score of 300.
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The highest possible score without any strikes results from ten frames of 9-1 spares followed by an additional 9 in the eleventh frame, corresponding to a pinfall of 19 each frame and a total score of 190.
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<td>10/-</td>
<td>9/1</td>
<td>9/1/9</td>
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</table>

These two games have identical frames with different orderings. There are four strikes in each game.
Angle of Attack

The shot should make contact with the pocket, which is the space between Pin 1 and Pin 3. An angle of six degrees with respect to the lane boards is considered optimal for the generation of strikes.²

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- A curved path requires the ball to experience **acceleration**. What forces can produce such an acceleration?
Suppose a ball (radius $R$) is rolling and sliding with velocity $v_C$ and angular rotation speed $\omega_C$.

The velocity of the contact point $B$ is given by:

$$v_B = v_C - \omega_C R$$

If $v_B = 0$, then the ball is undergoing pure rolling. Otherwise, the ball is sliding and frictional forces will act on the ball.
A Rolling/Sliding Ball

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The Effect of Friction

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- Energy is destroyed when $v_C$ and $\omega_C$ have opposite signs.
- Excess rotational energy can be thought of as a power source for the translational motion. The ball can accelerate as long as there is excess spin.
- Conversely, insufficient rotational energy will result in a power drain on the translational motion and the ball will decelerate until a balance is reached.
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Practical Observations

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- Theoretically, if enough backspin were generated, the ball could actually reverse its direction of motion and come back towards the bowler.
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For the typical bowler, the ball is released with a good balance of rotational and translational energy and the ball will generally achieve a pure rolling state before reaching the pins.
Mathematical Model, Part I

- Physical parameters:
  - Mass $= m$
  - Ball Radius $= r$
  - Rotational Inertia $= I = \frac{2}{5}mr^2$
  - Coefficient of Friction $= \mu$
  - Gravitational Constant $= g$

Oil conditions: Although oil patterns vary from one place to the next, typically a bowling lane is oiled from the foul line to a point about fifteen feet in front of the pins. This last section has a much higher coefficient of friction due to the lack of oil and often contributes a noticeable redirection of the ball trajectory. (the "extra" spin must last long enough for this to work.)
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Mathematical Model, Part II

Variables: We will use a vector
\[ x = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 & x_5 & x_6 \end{pmatrix} \]
- \( x_1 \) = position in lane (from left)
- \( x_2 \) = position along lane (from foul line)
- \( x_3 \) = velocity in lane (left to right)
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- **System of differential equations**: One obtains a linear system of first order differential equations by applying Newton’s 2nd law to the bowling ball. (Both translational and rotational forms.)
Mathematical Model, Part III

Friction Forces:
\[ F_{x_1} = -\mu m g \operatorname{sgn}(v_{x_1}) \]
\[ F_{x_2} = -\mu m g \operatorname{sgn}(v_{x_2}) \]

Newton's second law:
\[ m \frac{d^2x_3}{dt^2} = F_{x_1} \]
\[ m \frac{d^2x_4}{dt^2} = F_{x_2} \]
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Rotational form of Newton’s second law:

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\frac{d x_6}{dt} = F_{x_1} r \\
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The complete system of differential equations:

\[
\begin{align*}
\frac{dx_1}{dt} &= x_3 \\
\frac{dx_3}{dt} &= F_{x_1} \\
\frac{dx_5}{dt} &= F_{x_2} r \\
\frac{dx_2}{dt} &= x_4 \\
\frac{dx_4}{dt} &= F_{x_2} \\
\frac{dx_6}{dt} &= -\frac{F_{x_1} r}{l}.
\end{align*}
\]
Rotational form of Newton’s second law:

\[ \frac{l}{dt} \frac{dx_6}{dt} = F_{x_1} r \]

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The complete system of differential equations:

\[ \frac{dx_1}{dt} = x_3 \]
\[ \frac{dx_2}{dt} = x_4 \]
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\[ \frac{dx_4}{dt} = \frac{F_{x_2}}{m} \]
\[ \frac{dx_5}{dt} = \frac{F_{x_2} r}{l} \]
\[ \frac{dx_6}{dt} = -\frac{F_{x_1} r}{l} \]

Notice that the components of acceleration for the ball are piecewise constant.
Implications of the Model

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- The ball experiences constant acceleration when it is sliding, which makes it possible for the ball to follow a parabolic path.

\[
\text{Parabolic trajectory: } f(x) = ax^2 + bx + c, \quad a = \frac{\tan(\pi/30)}{60}, \quad b = -\tan(\pi/30), \quad c = 1.75; \quad x \text{ in feet}
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It is possible to achieve the six degree angle with a pure parabola: \( f(x) = ax^2 + bx + c, \ a = \tan \left( \frac{\pi}{30} \right)/60, \ b = -\tan \left( \frac{\pi}{30} \right), \ & c = 1.75; \ x \text{ in feet} \)
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Cheating the Geometry

- The model allows us to compare trajectories obtained for various spins, speeds, and release angles.
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1.60 degrees
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The three trajectories on the previous slide illustrate three stages of my bowling game:

- Straight balls: Throwing straight with a house ball.
- No-thumb spin: Creating spin by leaving the thumb out and torquing a house ball.
- Fingertip Grip: Creating spin with a personalized, fingertip grip ball.
The Three Stages

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<th>Strike %</th>
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<td>120</td>
<td>17%</td>
</tr>
<tr>
<td>No-thumb</td>
<td>130</td>
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<td>Fingertip</td>
<td>160</td>
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The Three Stages
# Straight Bowling

- **40 games:**
  - **Average:** 120
  - **First-Ball:** 7.1

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No-Thumb Spin Bowling

- 100 games:

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<tr>
<td>130</td>
<td>7.6</td>
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</table>

7/27/2004  Tropicana Lanes  40  7  3  10  7  3  10  10  10  9  1  10  9  1  10  8  0
CF: 10, STR: 6  20  40  60  90  119  139  159  179  199  217
Fingertip Grip

60 games:

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The dynamics of the bowling ball, the oil, the gutters, etc. exhibit interesting concepts from math and physics that are accessible to anyone with a background in differential equations.