A Mathematical Glance at Zombie Infestation

Brody Dylan Johnson

April 21, 2010

Reference: "When Zombies Attack!: Mathematical Modelling of an Outbreak of Zombie Infection" by Huden, Imad, Munz, and Smith (2009).

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Night of the Living Dead (1968) - Trailer

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28 Days Later (2002) - Unofficial Trailer

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Comparison of Zombie Portrayal

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Night of the Living Dead:

Comparison of Zombie Portrayal

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- Radioactive exposure causes zombiism, but the condition can infect others upon exposure.

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28 Days Later:

- Zombiism is caused by a viral infection (contagious) which originated in a laboratory experiment involving primates.

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- Zombiism is caused by a viral infection (contagious) which originated in a laboratory experiment involving primates.
- Zombies are quick and demonstrate some cunning, but avoid daylight.

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Dungeons & Dragons (RPG)

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Zombie

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Description: Animated corpes which move slowly, but are relentless in their attack.

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Description: Once human creatures with terrible cunning that feed on human corpses. Victims transform into another ghoul.

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Reference/Image Credits: Advanced Dungeons & Dragons, *Monster Manual* (1979).

A Simple Mathematical Model:

Notation:

- $Z \equiv$ number of zombies

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Assumptions:

A Simple Mathematical Model:

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Notation:

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Assumptions:

- Encounters between zombies and humans occur at a rate proportional to both Z and H.

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Assumptions:

- Encounters between zombies and humans occur at a rate proportional to both Z and H.
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- In the absence of zombies the human population grows exponentially.

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- In the absence of humans the zombie population decays exponentially.

A Predator-Prey Model:

The above assumptions lead to the standard predator prey model:

$$\frac{dH}{dt} = a_1 H - a_2 H \cdot Z$$

$$\frac{dZ}{dt} = -a_3 Z + a_4 H \cdot Z,$$

where $a_1, a_2, a_3, a_4 > 0$.

This is equivalent to

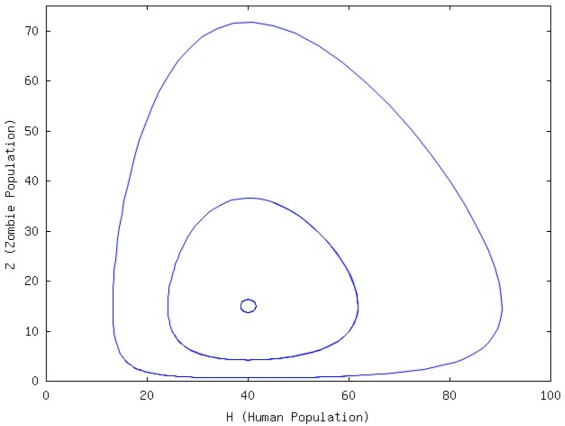
$$\frac{dH}{dt} = a_2 H(r_1 - Z)$$

$$\frac{dZ}{dt} = a_4 Z(H - r_2),$$

where $r_1 = \frac{a_1}{a_2}$ and $r_2 = \frac{a_3}{a_4}$.

The system has equilbria at $(H, Z) = (0, 0)$ and $(H, Z) = (r_2, r_1)$.

Sample Trajectories:



Sample Trajectories:

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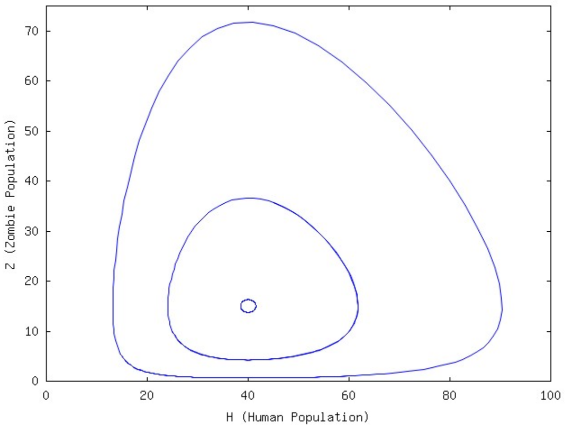

OBSERVATION: This model predicts a natural balance between zombies and humans with greater fluctuation farther from the equilibrium.

A Reality Check:

- Zombie infestations are generally portrayed as occurring very rapidly. Therefore, natural growth/decay rates of both human and zombie species can effectively be ignored.

A Reality Check:

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- Humans make reluctant prey. Most, if not all, fictional accounts of zombie infestation include human resistance. This can be included in the model by introducing a new variable to represent a militia of sorts:

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  - $Z \equiv$ number of zombies
  - $H \equiv$ number of civilian humans
  - $M \equiv$ number of militant humans

Building a New Model:

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- The number of militants decays at a rate proportional to the number of zombie-militant encounters.

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The New Model:

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$$\frac{dH}{dt} = -a_1 H \cdot Z$$
$$\frac{dM}{dt} = -a_2 M \cdot Z$$
$$\frac{dZ}{dt} = a_3 H \cdot Z - a_4 M \cdot Z$$

where $a_1, a_2, a_3, a_4 > 0$.

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- Equilibrium points:
  - Doomsday equilibrium: $(H, M, Z) = (0, 0, Z_0)$, $Z_0 > 0$.

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- Equilibrium points:
  - Doomsday equilibrium: $(H, M, Z) = (0, 0, Z_0)$, $Z_0 > 0$.
  - Eradication equilibrium: $(H, M, Z) = (H_0, M_0, 0)$, $H_0, M_0 > 0$.

Classifying the Equilibria:

- The Jacobian (matrix of partial derivatives) is given by

$$J(H, M, Z) = \begin{bmatrix} -a_1 Z & 0 & -a_1 H \\ 0 & -a_2 Z & -a_2 M \\ a_3 Z & -a_4 Z & a_3 H - a_4 M \end{bmatrix}.$$

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  - Eradication equilibrium $(H_0, M_0, 0)$: $\lambda_1 = 0$, $\lambda_2 = 0$, & $\lambda_3 = a_3 H - a_4 M$.
- The most important quantity is $a_3 H - a_4 M$.

Doomsday scenario:

Parameters:

$$a_1 = 0.08, \; a_2 = 0.02, \; a_3 = 0.01, \; a_4 = 0.05$$

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Parameters:

$$a_1 = 0.08,\ a_2 = 0.02,\ a_3 = 0.01,\ a_4 = 0.05$$

Initial Conditions:

$$H(0) = 100,\ M(0) = 5,\ Z(0) = 25$$

Doomsday Simulation

Eradication scenario:

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Parameters:

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Initial Conditions:

$$H(0) = 80,\ M(0) = 25,\ Z(0) = 25$$

Eradication Simulation

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Concluding Remarks:

- Model parameters play a crucial role in the limiting behavior of solutions.

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- Model parameters play a crucial role in the limiting behavior of solutions.
- The models in the paper “When Zombies Attack!...” address infection, incubation, and recovery of zombiism by considering appropriate subsets of the population.
- The most common form of zombiism is cured by the end of a lecture.

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The End!