DISCRETE MATH: LECTURE 2

DR. DANIEL FREEMAN

1. Chapter 2.1 Logical Form and Logical Equivalence

1.1. Deductive Logic.

- An **Argument** is a sequence of statements aimed at demonstrating the truth of an assertion.
- The assertion at the end of the sequence is called the **Conclusion**, and the preceding statements are called **Premises**.
- To illustrate the logical form of arguments, we use letters of the alphabet (such as p, q, and r) to represent the component sentences of an argument.

1.2. Statements and Truth Tables.

- A **Statement** (or **Proposition**) is a sentence that is true or false but not both.
  For example: Two plus two equals four.
  For example: Two plus two equals five.
  For example: $x + y > 0$.
- If sentences are to be statements, they must have well-defined **Truth Values**–they must either be true or false. We can use a truth table to summarize truth values.
- If p is a statement variable, the **negation** of p is "not p" or "It is not the case that p" and is denoted $\sim p$. It has opposite truth value from p; if p is true, $\sim p$ is false; if p is false, $\sim p$ is true.

p	$\sim p$
T	
F	

- If p and q are statement variables, the **conjuction** of p and q is "p and q," denoted $p \wedge q$. It is true when, and only when, both p and q are true. If either p or q is false, or if both are false, $p \wedge q$ is false.

p	q	$p \wedge q$
T	T	
T	F	
F	T	
F	F	

- If p and q are statement variables, the **Disjunction** of p and q is "p or q," denoted $p \vee q$. it is true when either p is true, or q is true, or both p and q are true; it is false only when both p and q are false.

- NOTE: The use of "or" in mathematics refers to the inclusive sense of the word. If you want to use the exclusive meaning you need to express "p or q but not both".

p	q	$p \vee q$
T	T	
T	F	
F	T	
F	F	

For example: Write down the truth table for $(p \vee q) \wedge \sim (p \wedge q)$.

p	q	$p \vee q$	$p \wedge q$	$\sim (p \wedge q)$	$(p \vee q) \wedge \sim (p \wedge q)$
T	T				
T	F				
F	T				
F	F				

1.3. **In Class Group Work.** Section 2.1, Page 37: Answer all of question 6, 8 parts a and d, 16, and 18.

2. Logical Equivalence

- Two *statement forms* are called **logically equivalent** if, and only if, they have identical truth values for each possible substitution of statements for their statement variables. The logical equivalence of statement forms P and Q is denoted by writing $P \equiv Q$.
- Two *statements* are called **logically equivalent** if, and only if, they have logically equivalent forms when identical component statement variables are used to replace identical component statements.
  For example: $\sim(\sim p) \equiv p$

p	$\sim p$	$\sim(\sim p)$
T		
F		

For example: $\sim(p \wedge q)$ is not logically equivalent to $\sim p \wedge \sim q$

p	q	$\sim p$	$\sim q$	$p \wedge q$	$\sim(p \wedge q)$	$\sim p \wedge \sim q$
T	T					
T	F					
F	T					
F	F					

2.1. **In Class Group Work.** Is $\sim(p \wedge q)$ logically equivalent to $\sim p \vee \sim q$?

- The negation of an *and* statemen is logically equivalent to the *or* statement in which each component is negated.
- The negation of an *or* statement is logically equivalent to the *and* statement in which each component is negated.
- These are called **De Morgan's Laws** and they are MUY IMPORTANTE!

2.2. **Tautologies and Contradictions.**

- A **Tautology** is a statement form that is always true regardless of the truth values of the individual statements substitued for its statement variables. A statement whose form is a tautology is a **tautological statement**. (i.e., you always write a "T" for a tautology in your truth table; a tautology will produce "all T's")
- A **Contradition** is a statement form that is always false regardless of the truth values of the individual statements substituted for its statement variables. A statement whose form is a contradiction is a **contradictory statement**. (i.e., you always write a "F" for a contradiction in your truth table; a contradiction will produce "all F's")
- Check out Theorem 2.11 on page 35 in section 2.1.

2.3. **In Class Group Work.** Use a truth table to show that $p \vee \sim p$ is a tautology and that $p \wedge \sim p$ is a contradiction.