DISCRETE MATH: LECTURE 8

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1. Chapter 4.2 Direct Proof and Counterexample 2: Rational Numbers

Definition. A real number r is **rational** if and only if it is equal to the quotient of two integers with a nonzero denominator. A real number is **irrational** if and only if it is not rational.

- We denote the set of rational numbers by $\mathbb{Q}$.
- Symbolically we can write, if $r \in \mathbb{R}$ then

$$r \in \mathbb{Q} \Leftrightarrow \exists a, b \in \mathbb{Z} \text{ such that } r = \frac{a}{b} \text{ and } b \neq 0.$$

(1) Is 0 a rational number?

(2) Is .123 a rational number?

(3) Is .123123123... a rational number?

Theorem 1.1 (4.2.2)**.** *The sum of two rational numbers is rational*

Proof. :

□

- In proving new results, we can use previously proven theorems. A theorem whose proof is easily given by using a previous theorem is called a corollary.

Corollary 1.2. *The double of a rational number is rational.*

Proof. :

□

In class group work, prove the following statements:

(1) The difference of any two rational numbers is rational.

(2) The average of any two rational numbers is rational.

(3) $\forall n \in \mathbb{Z}$, if n is odd then $n^2 + n$ is even.

2. 4.3 Direct Proof and Counterexample 3: Divisibility

Definition. If n and d are integers and $d \neq 0$ then n is **divisible by** d if and only if n equals d times some integer.

"n is divisible by d" is also written as:

- n is a multiple of d
- d is a factor of n
- d is a divisor of n
- d divides n

- We denote n is divisible by d with $d|n$
- Symbolically we can write, if $n, d \in \mathbb{Z}$ and $d \neq 0$ then

$$d|n \Leftrightarrow \exists k \in \mathbb{Z} \text{ such that} n = dk$$

(1) Is 21 divisible by 7?

(2) Does $5|100$?

(3) If $n \in \mathbb{Z}$ and $n \neq 0$ then does $n|n$?

(4) If $n \in \mathbb{Z}$ and $n \neq 0$ then does $n|0$?

Theorem 2.1. *For all positive integers a and b, if $a|b$ then $a \leq b$.*

Theorem 2.2. *The only divisors of 1 are 1 and -1.*

Theorem 2.3 (Unique factorization of Integers Theorem)**.** *Given any integer $n > 1$, there exists a unique positive integer k, distinct prime numbers $p_1 < p_2 < ... < p_k$, and positive integers $e_1, e_2, ..., e_k$ such that*

$$n = p_1^{e_1} p_2^{e_2} \cdots p_k^{e_k}.$$

and this expression is unique. This is called the standard factored form for n.

- In class group work, prove the following:

(1) For all integers a, b, and c, if a divides b and b divides c then a divides c.

(2) For all integers a, b, and c, if a divides b and a divides c then a divides $b + c$.

(3) The sum of any three consecutive integers is divisible by 3.