Homework 4

1. Let R be a PID and let $a, b \in R$. Show that the greatest common divisor d of a and b exists in R and $d = ax + by$ for some $x, y \in R$.

2. Let R be a noetherian UFD and suppose that whenever $a, b \in R$ are not both zero and have no common prime divisor, there exist elements $u, v \in R$ such that $au + bv = 1$. Show that R is a PID.

3. An ideal P in a commutative ring R is called primary if for all $x, y \in R$, $xy \in P$, $x \notin P$ implies $y^n \in P$ for some positive integer n. Show that if P is a primary ideal then $\sqrt{P} = \{x \in R : x^n \in P \text{ for some } n > 0\}$ is a prime ideal of R.