Homework-2

1. Show that an infinite group is cyclic if and only if it is isomorphic to each of its proper subgroups.

2. Show that a group in which all the mth powers commute with each other and all the nth powers commute with each other, m and n relatively prime, is abelian.

3. Prove that if a group G is finite, then the Frattini subgroup $\phi(G)$ is the set of nongenerators of G. (see page 27, problems 2.5 and 2.7 in Isaacs for definitions of maximal subgroup, Frattini subgroup and nongenerators).

4. Show that there does not exist any nonzero group homomorphism from the group S_3 to the group $\mathbb{Z}_3$.

5. Let a be an element of a group G such that $o(a) = r$. Let m be a positive integer. Prove that $o(a^m) = \frac{r}{(m,r)}$.